Journal of Advanced Research in Instrumentation and Control Engineering
Volume 7, Issue 3&4 - 2020, Pg. No. 1-4
Peer Reviewed Journal

Research Article

Design of Controller for Large-Scale Uncertain Systems Adopting Affine Arithmetic

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DOI: https://doi.org/10.24321/2456.1398.202002

I N F O

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How to cite this article:
Dora HM, Narasimhulu T, Pasumarthi MR. Design of Controller for Large-Scale Uncertain Systems Adopting Affine Arithmetic. *J Adv Res Instru Control Engg* 2020; 7(3&4): 1-4.

Date of Submission: 2020-10-23
Date of Acceptance: 2020-11-13

A B S T R A C T

In this article presents a new controller design technique for large scale uncertain systems. The new controller is designed through a reduced order system from certain high order system. In the projected reduction method, the numerator coefficients are obtained with γ-table while the denominator polynomial is achieved with δ - table. An optimised reduced order model is derived with minimum value of ISE. In this proposed method gives better stability of the reduced order model, if consider the original stable high order system. A PID controller is designed for the high order original systems through its proposed reducer order model. Consider several numerical examples are available in the literatureto illustrate the usefulness of the proposed method andit gives sufficientoutcome.

Keywords: Affine Arithmetic, Order Reduction, Uncertain Systems, Controller

Introduction

Basically, the higher order system is very difficult to analyse, design and control. Due to this reasons higher order model can be represent in the nature of lower order modelfrom original higher order model. The analysis and design problem are appeared due to connection of uncertainty to changingextent in the science and technology. The type of uncertainties are begin in the system may be concerned by data problems. For example: disappeared or inaccessible data, data might be present but unpredictable or uncertain due to computation errors, the representation of the data may be inaccurate or unpredictable, etc. Uncertainty can be described in the number of types, like probabilistic, rounded or else fuzzy descriptions. However, in many systems the factors are constant but, in some cases, uncertain within a predetermined range this type of systems iscalled interval systems.[1-4]

The major advantage of Interval Arithmetic (IA) is to design and study of possible uncertainties in magnitude and to control internal estimation errors like rounding errors and the disadvantage of interval arithmetic is influence to be conventional process or more conservative: The interval gives much comprehensive than the actual range of the corresponding quantities, often to the point of ineffectiveness. This problem is exceptionally severe in long computing sequence; here the intervals are calculated through one input stage used as the next stage.[5-7]

The main reason for selecting affine arithmetic is to advance the dependency problem, which occurs especially in the range evaluation of a function. Affine arithmetic is probably used for every numerical problem, where essential assured enclosures to continuous functions, such as resolving of non-linear equations, dynamical systems analysing, differential equations and integrating functions, etc. The applications of Affine arithmetic are rays tracing, curves designing, parametric surfaces, range analysis, dependency problem, process control, electric circuits analysis etc. Especially Affine arithmetic can be represented in the form of an enhancement of interval arithmetic , and it is similar to stable inthe established interval arithmetic, Taylor arithmetic in the first-order, the close on-slope model, and ellipsoidal calculation in the sense that it is an self-regulating method to develop first-order standard approximations to the ordinary equation.[9-10]

The proposed method to overcome the problem of interval arithmetic by using some computational models and it is maintained correlations amid input and computed quantities of first–order system, these correlations are accordingly exploited in original operations, with the result that in many cases affine arithmetic is able to produce interval approximation that are much better than the ones obtained with standard interval arithmetic. Moreover, affine arithmetic also essentially provides a geometric representation for the combined range of related quantities that can be used to increase the efficiency of interval methods. Like interval arithmetic, affine arithmetic gives the round-off errors and transition errors by automatically and affecting each calculated size.[9-11] Then an optimal model (with minimum ISE) is achieved by changing the interpolation points. By adoptingAffine Arithmetic a PID controller is performed for the given original high order model.

PID controllers are suitable to many control system applications, and execute acceptably without any advancement or only low tuning and no direct information of the process and thus overall performance is sensible. This controller is the best controller in an observer and it gives better performance can be obtained by overtly modeling the character of the process without modernize to an observer. PID controllers are used alone, it can give poor performance and the PID loop gains must be reduced so that the control system does not overshoot, oscillate or hunt about the control set point value. They also have difficulties in the presence of non-linearity's, may trade-off regulation versus response time, do not respond to varying process performance and have interval in responding to wide interruption.

Reduction Procedure

In this section, the proposed method for reducing higher order is interpreted. The reduced order coefficients are accumulated by applying Modified Routh Approximation Method,[1] and without considering time instants of the original higher order model. In this suggested method required Gamma and Delta table is calculated from the following equations.

Consider thetransfer function of higher order interval system model as follows:

$$G(s) = \frac{[d_1^-, d_1^+]s^{n-1} + [d_2^-, d_2^+]s^{n-2} + \cdots + [d_n^-, d_n^+]}{[c_0^-, c_0^+]s^n + [c_1^-, c_1^+]s^{n-1} + \cdots + [c_n^-, c_n^+]} \quad (1)$$

The reduced rth ordersystem $R_r(s)$ is described by

$$R_r(s) = \frac{D_r(s)}{C_r(s)} \quad (2)$$

Where, $C_r(s)=s^2\, C_{r-2}\,(s) + [\gamma_r^-, \gamma_r^+]C_{r-1}\,(s)$

$$D_r\,(s) = [\delta_r^-, \delta_r^+]s^{r-1} + s^2\, D^{r-2}\,(s) + [\gamma_r^-, \gamma_r^+]\, D_{r-1}\,(s) \quad (3)$$

Design Procedure

Steps

- Identify the transfer function of closed loop system (T(S)) from controlled system ($G_c(s)$) for which the constants are to be determined for a given system.
- Determine the constants from characteristic equation for the closed-loop as per the ITAE performance index optimum coefficient for minimum ITAE for step input is given table.

The general closed loop transfer function for the zero steady state step error system, as

Table 1.Optimum forms of the transfer function of closed loop system by ITAE criterion (zero steady – state step error system)

$T(s) = \frac{C(s)}{R(s)} = \frac{a_n}{s^n + a_1 s^{n-1} + \ldots + a_{n-1}s + a_n}$
$s + \omega_n$
$s^2+1.4\omega_n s + \omega_n^2$
$s^3+1.75\omega_n s^2+2.15\omega_n^2 s + \omega_n^3$
$s^4+2.1\omega_n s^3+3.4\omega_n^2 s^2+2.7\omega_n^3 s + \omega_n^4$

Table 2.Optimum forms of the transfer function of closed loop system based on the ITAE criterion (zero steady – State Ramp Error system)

$T(s) = \frac{C(s)}{R(s)} = \frac{a_{n-1}s + a_n}{s^n + a_1 s^{n-1} + \ldots + a_{n-1}s + a_n}$
$s^2+3.2\omega_n s + \omega_n^2$
$s^3+1.75\omega_n s^2+3.25\omega_n^2 s + \omega_n^3$
$s^4+2.41\omega_n s^3+4.93\omega_n^2 s^2+5.14\omega_n^3 s + \omega_n^4$

- From the above step we can write T(S) in form of known coefficient except for ω_n. From these we can get the constants of $G_c(S)$ in the terms of ω_n.
- For the specified setting time, choose a suitable value of and then find ω_n.
- Now full $G_c(S)$ is known as so in T(S).
- Presence of zeros of $G_c(S)$ in T(S) normally does not allow the % overshoot requirement to be met.
- By determining a pre-filter $G_p(S)$, we can eliminate the zeros of T(S).

ISSN: 2456-1398
DOI: https://doi.org/10.24321/2456.1398.202002

Case Study

Consider the 4th order system as follows:

$$G(s) = \frac{d_n(s)}{e_n(s)}$$

$d_n(s) = [6.2164,1]s^3 + [4.6146,6]s^2 + [1.7134,11]s + [0.25,6]$

$e_n(s) = [73.018,1]\ s^4 +’\ [50.03,17]s^3 + [17.104,82]s^2 + [1.919,130]s + [0.25,100]$

A Second order reduced model is calculated from given higher order system, by following steps, adopting the suggested method mentioned in the section-II.

$$R_2(s) = \frac{a_k(s)}{b_k(s)}$$

By normalizing the above transfer function, then we get the reduced order denominator as (based on routh method):

$b_k(s) = [0.9, 1]\ s^2 + [0.07881, 1.4409]s + [0.0186, 1.3449]$ and

Reduced order numerator (based on modified routh approximation method):

$$R_2(s) = \frac{[0.05244,0.1205]s+[0.0194,0.0806]}{[0.9,1]s^2+[0.07881,1.4409]s+[0..0186,1.3449]}$$

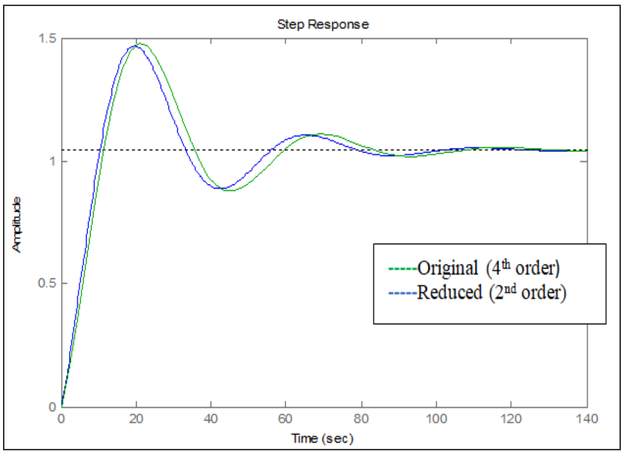

Figure 1.Step response of original 4th order and reduced 2nd order system. (upper limit)

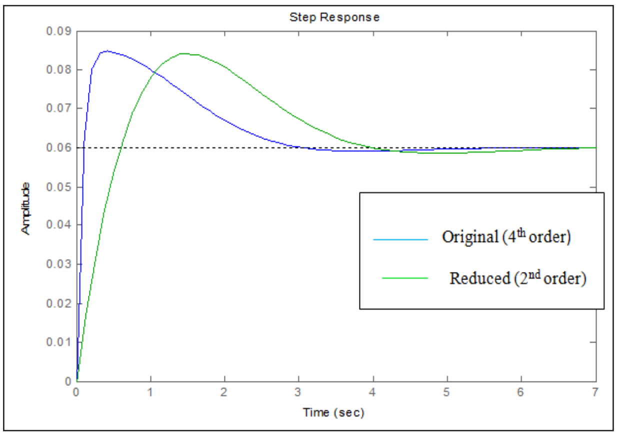

Figure 2.Step response of original 4th order and reduced 2nd order system. (lower limit)

Design Steps of Reduced Order Model for PID Controller

Consider PID controller transfer function as,

$$G_c(s) = \frac{k_d s^2 + k_p s + k_i}{s}$$

Applying ITAE performance index method to the reduced model, then of Kp, Ki and Kdvalues are obtained

$$R_2(s) = \frac{0.05244s + 0.0194}{s^2 + 0.07881s + 0.0186}$$

Then get the reduced order closed-loop transfer function for the tuned PID values are

$$K_p = -9.1331, \quad K_i = 7.654\,, \quad K_d = 23.899$$

Comparing the characteristic equations of compensated system and the optimum ITAE system as,

$$s^3 + 1.75\omega_n s^2 + 3.25\omega_n^2 s + \omega_n^3$$

The PID controller is added in the forward path then to obtain the transfer function of closed loop system with negative unity feed back as follows:

$$T_c(s) = \frac{G_c(S)G(S)}{1 + G_c(S)G(S)}$$

Where $G(s)$ is the high order system and $G_c(s)$ is the transfer function of PID controller.

Applying ITAE performance index method to the reduced model, then obtain the Kp, Ki and Kd values

$$R_2(s) = \frac{0.1205s + 0.0806}{s^2 + 1.4409s + 1.3449}$$

Now calculatethe reduced orderclosed-loop transfer function for the tuned PID values are

$K_p = 0.889$, $K_i = 133.33$, $K_d = 19.183$

Comparing the characteristic equations of compensated system and the optimum ITAE system as,

$s^3 + 1.75\omega_n s^2 + 3.25\omega_n^2\ s + \omega_n^3$

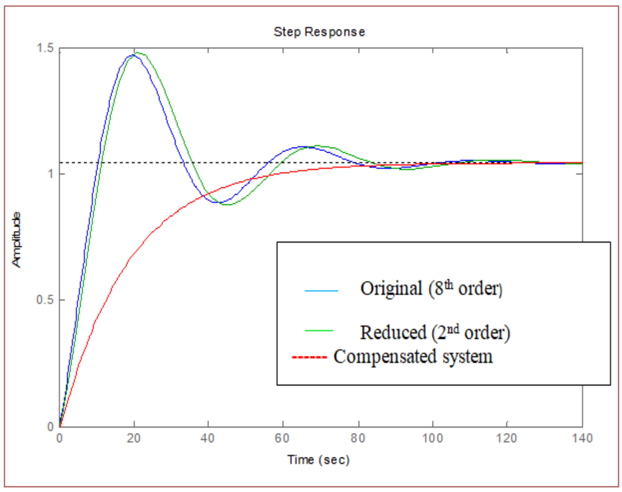

Figure 3.Step response of compensated system and uncompensated systems

ISSN: 2456-1398
DOI: https://doi.org/10.24321/2456.1398.202002

The PID controller is added to the forward path and the closed loop transfer function with unity feedback of the system is given as:

$$T_c(s) = \frac{G_C(S)G(S)}{1 + G_C(S)G(S)}$$

Where G(s) is the high order system and $G_c(s)$ is the PID controller transfer function.

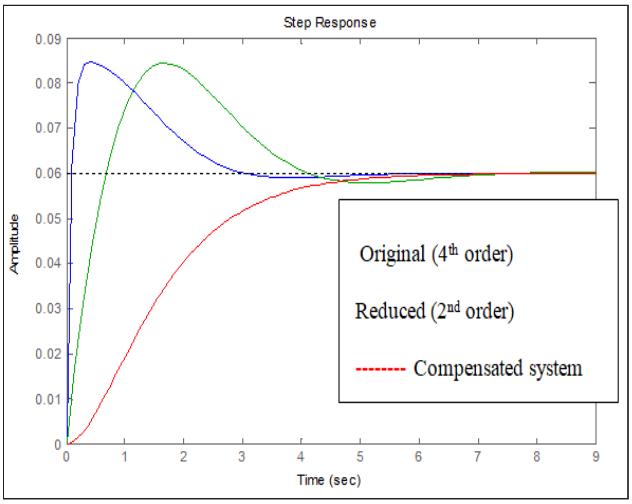

Figure 4.Step response of compensated system and uncompensated systems

Conclusion

The analysis and designofcontroller for this type of systems become complexity,flexibility and costly. The PID controller is the most commonly used in control strategy. There are many difficult methods to find suitable parameters of the controllers. The proposed Affine Arithmetic methodhas become a better appliancefor analysis and simulation of high-order uncertain systems. Due to these reasons, it is better to performfrom high –order system to lower –order model without allowing the main features of the original system. The developed lower- order system from the original system characterize the performance of the original system is very costly. So, Model order reduction method is applied for simulation of higher-order systems. Affine Arithmetic maintain the correlations between those quantities and its providedbetter steady intervals than interval arithmetic.

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ISSN: 2456-1398
DOI: https://doi.org/10.24321/2456.1398.202002